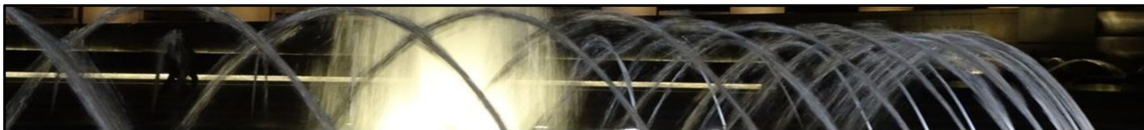

Algebra 2
Chapter 3
Solve Quadratic Equations



- This Slideshow was developed to accompany the textbook
  - *Big Ideas Algebra 2*
  - *By Larson, R., Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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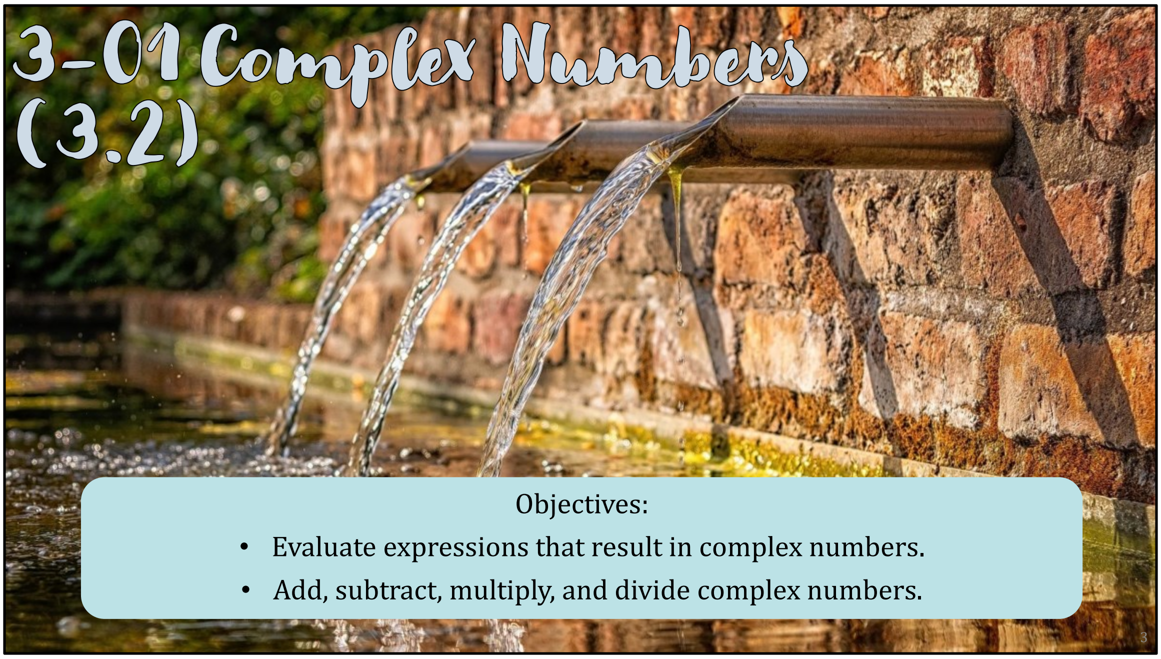
3-01 Complex Numbers (3.2)
Objectives:
• Evaluate expressions that result in complex numbers.
• Add, subtract, multiply, and divide complex numbers.
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3-01 Complex Numbers (3.2)

- **Imaginary Number** (imaginary unit)
  - i
  - $i = \sqrt{-1}$
  - $i^2 = -1$
- **Complex Number**
  - $a + bi$
  - a is real part
  - bi is imaginary part
  - Any number with i is called imaginary
- $\sqrt{-9}$
- $\sqrt{-12}$

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$$\sqrt{9}\sqrt{-1} = 3i$$

$$\sqrt{4}\sqrt{3}\sqrt{-1} \rightarrow 2\sqrt{3}i$$

3-01 Complex Numbers (3.2)

- **Adding and Subtracting Complex Numbers**
  - Add the same way you add $(x + 4) + (2x - 3) = 3x+1$
  - Combine like terms
- Simplify
  - $(-1 + 2i) + (3 + 3i)$
  - $(2 - 3i) - (3 - 7i)$
  - $2i - (3 + i) + (2 - 3i)$

$2 + 5i$
$-1 + 4i$
$-1 - 2i$

3-01 Complex Numbers (3.2)

- **Multiplying complex numbers**
  - FOIL
  - Remember $i^2 = -1$
- Multiply
  - $-i(3 + i)$
- $(2 + 3i)(-6 - 2i)$
- $(1 + 2i)(1 - 2i)$

$-3i - i^2 = -3i - (-1) = 1 - 3i$

$-12 - 4i - 18i - 6i^2 = -12 - 22i - 6(-1) = -6 - 22i$

$1 - 2i + 2i - 4i^2 = 1 - 4(-1) = 5$

3-01 Complex Numbers (3.2)

- Notice on the last example that the answer was just real
- **Complex conjugate** → same numbers just opposite sign on the imaginary part
  - When you multiply complex conjugates, the product is real

- **Dividing Complex Numbers**
  1. To divide, multiply the numerator and denominator by the complex conjugate of the denominator
  2. No imaginary numbers are allowed in the denominator when simplified

3-01 Complex Numbers (3.2)

- Divide
- $\frac{2-7i}{1+i}$
- $\frac{2i}{2-i}$

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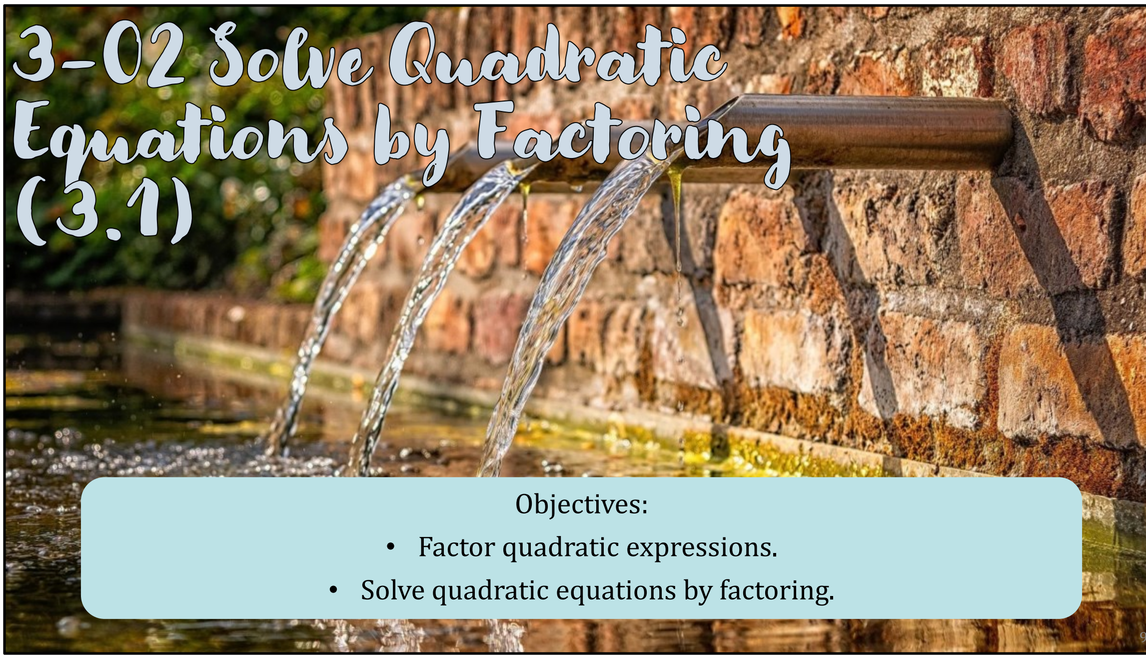
$$\frac{2-7i}{1+i} = \frac{(2-7i)}{(1+i)}\frac{(1-i)}{(1-i)} = \frac{2-2i-7i+7i^2}{1-i+i-i^2} = \frac{2-9i+7(-1)}{1-(-1)} = \frac{-5-9i}{2}$$
$$= -\frac{5}{2} - \frac{9}{2}i$$

$$\frac{2i}{2-i} = \frac{2i(2+i)}{(2-i)(2+i)} = \frac{4i+2i^2}{4+2i-2i-i^2} = \frac{4i+2(-1)}{4-(-1)} = \frac{-2+4i}{5} = -\frac{2}{5} + \frac{4}{5}i$$


3-02 Solve Quadratic Equations by Factoring (3.1)
Objectives:
• Factor quadratic expressions.
• Solve quadratic equations by factoring.

3-02 Solve Quadratic Equations by Factoring (3.1)

- Factoring is the opposite of FOILing

- Factoring undoes multiplication

- $(x + 2)(x + 5) = x^2 + 7x + 10$
  - $x + 2$ called **binomial**
  - $x^2 + 7x + 10$ called **trinomial**

3-02 Solve Quadratic Equations by Factoring (3.1)

- **Factor a Quadratic in the form of $ax^2 + bx + c$,**

1. Factor out any common terms first, then factor what's left
2. Write two sets of parentheses like (   )(   ).
3. Guess: Find two expressions whose product is ax^2 and put them at the beginning of each set of parentheses.
4. Guess: Find two expressions whose product is c and put them at the end of each set of parentheses. Pay attention + and – signs.
5. Check: Calculate the outers + inners and compare it to the middle bx.
    a. If the outers + inners = bx, then the factoring is correct.
    b. If the outers + inners = $-bx$ (the correct number but wrong sign), then change the signs in the parentheses.
    Otherwise, retry with new guesses.

3-02 Solve Quadratic Equations by Factoring (3.1)

- Factor
  - $x^2 - 3x - 18$
  - $n^2 - 3n + 9$
- $r^2 + 2r - 63$

12

$(x - 6)(x + 3)$

Cannot be factored

$(r + 9)(r - 7)$

3-02 Solve Quadratic Equations by Factoring (3.1)

- Factor
  - $14x^2 + 2x - 12$
- $3x^2 - 18x$

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$2(7x^2 + x - 6) \rightarrow 2(7x - 6)(x + 1)$

$3x(x - 6)$

3-02 Solve Quadratic Equations by Factoring (3.1)

- Factor
  - $2x^2 - 32$

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$2(x^2 - 16) \rightarrow 2(x - 4)(x + 4)$

3-02 Solve Quadratic Equations by Factoring (3.1)

- **Zero Product Property**
  - If $a \cdot b = 0$, then either a or b is 0.
- **Solve a Quadratic Equation by Factoring**

1. Make the quadratic expression equal 0.
2. Factor the quadratic expression.
3. Set each factor equal to zero as two separate equations.
4. Solve each equation.
5. Check your solutions

3-02 Solve Quadratic Equations by Factoring (3.1)

- Solve
  - $x^2 - x - 42 = 0$

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$x^2 - x - 42 = 0 \rightarrow (x - 7)(x + 6) = 0 \rightarrow$

$x - 7 = 0 \rightarrow x = 7$

$x + 6 = 0 \rightarrow x = -6$

Solutions are $x = -6, 7$

3-02 Solve Quadratic Equations by Factoring (3.1)

- Solve
  - $9t^2 - 12t + 4 = 0$
- $3x - 6 = x^2 - 10$

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$(3t - 2)^2 = 0 \rightarrow 3t - 2 = 0 \rightarrow 3t = 2 \rightarrow t = 2/3$

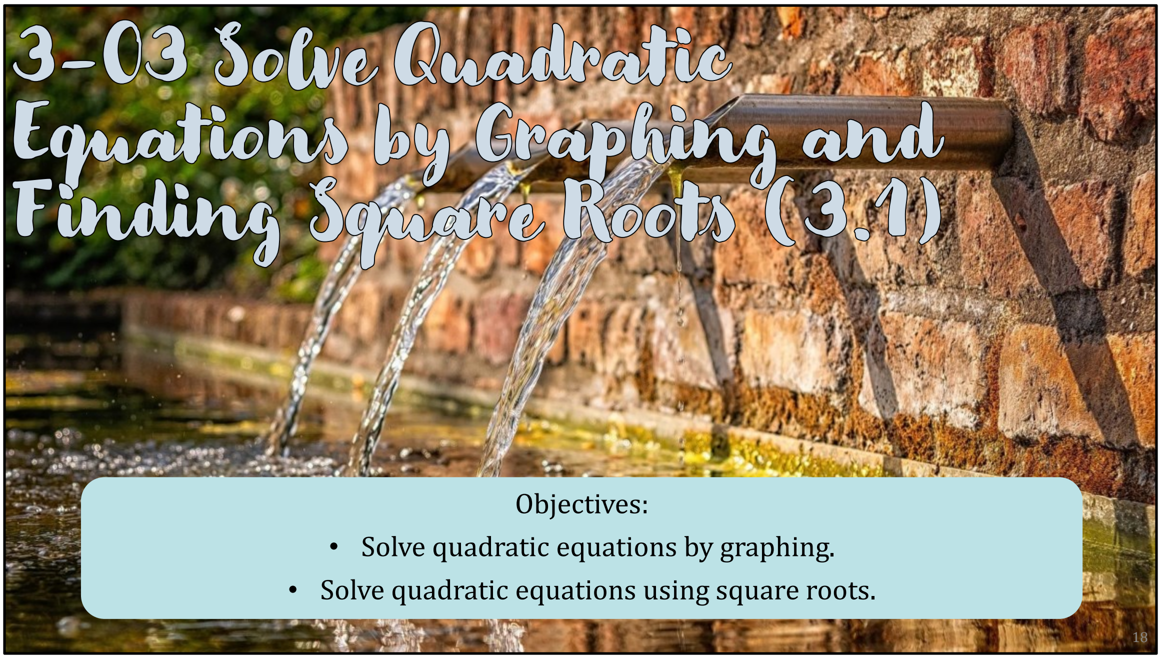
Put in standard form

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

$x - 4 = 0 \rightarrow x = 4$

$x + 1 = 0 \rightarrow x = -1$


3-03 Solve Quadratic Equations by Graphing and Finding Square Roots (3.1)
Objectives:
• Solve quadratic equations by graphing.
• Solve quadratic equations using square roots.
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3-03 Solve Quadratic Equations by Graphing and Finding Square Roots (3.1)

- **Solving Quadratic Equations by**
- ***Graphing***

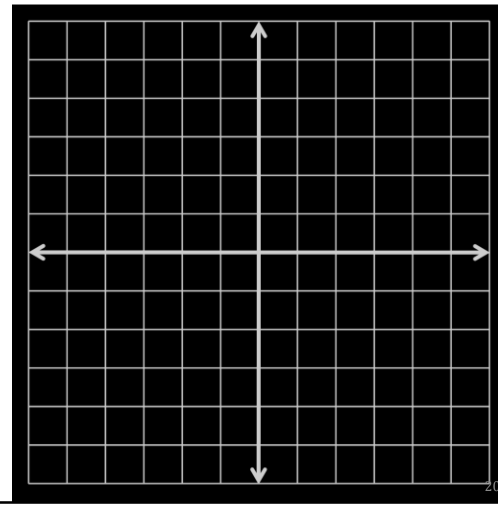
1. Make the equation equal zero.
2. Graph the equation.
3. Find the x-values of the x-intercepts.

- ***Square Roots***

1. Solve for the squared expression.
2. Take a square root. Remember to put $\pm$.
3. Finish solving for x.
4. Check your solutions.

3-03 Solve Quadratic Equations by Graphing and Finding Square Roots (3.1)

- Solve by graphing
- $x^2 - 2x - 3 = 0$



Graph
Solutions are *x*-intercepts
x = 3, x = −1

3-03 Solve Quadratic Equations by Graphing and Finding Square Roots (3.1)

- Solve by using square roots.
- $2x^2 + 14 = 70$
- $4x^2 + 20 = 16$

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$$2x^2 + 14 = 70$$
$$2x^2 = 56$$
$$x^2 = 28$$
$$x = \pm\sqrt{28} = \pm\sqrt{4 \cdot 7} = \pm 2\sqrt{7}$$

$$4x^2 + 20 = 16$$
$$4x^2 = -4$$
$$x^2 = -1$$
$$x = \pm\sqrt{-1} = \pm i$$

3-03 Solve Quadratic Equations by Graphing and Finding Square Roots (3.1)

- $\frac{3}{4}(x+1)^2 = 10$
- $2x^2 = 5x^2 + 24$

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$$\frac{3}{4}(x+1)^2 = 10$$
$$(x+1)^2 = \frac{40}{3}$$
$$x+1 = \pm\sqrt{\frac{40}{3}}$$
$$x+1 = \pm\frac{\sqrt{40}}{\sqrt{3}}$$
$$x+1 = \pm\frac{2\sqrt{10}}{\sqrt{3}} = \pm\frac{2\sqrt{10}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{2\sqrt{30}}{3}$$
$$x = -1 \pm \frac{2\sqrt{30}}{3}$$

$$2x^2 = 5x^2 + 24$$
$$-3x^2 = 24$$
$$x^2 = -8$$
$$x = 2\sqrt{2}i$$

3-03 Solve Quadratic Equations by Graphing and Finding Square Roots (3.1)

- A fruit stand charges \$3 per pound of apples and sells 20 pounds each day. They try dropping the price by \$0.50 and sell 5 more pounds a day. How much should the fruit stand charge to maximize their daily revenue? What is their maximum daily revenue?

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Revenue is price × number sold

$$R = p \times s$$
$$p = (2 - 0.1x)$$
$$s = (20 + 5x)$$

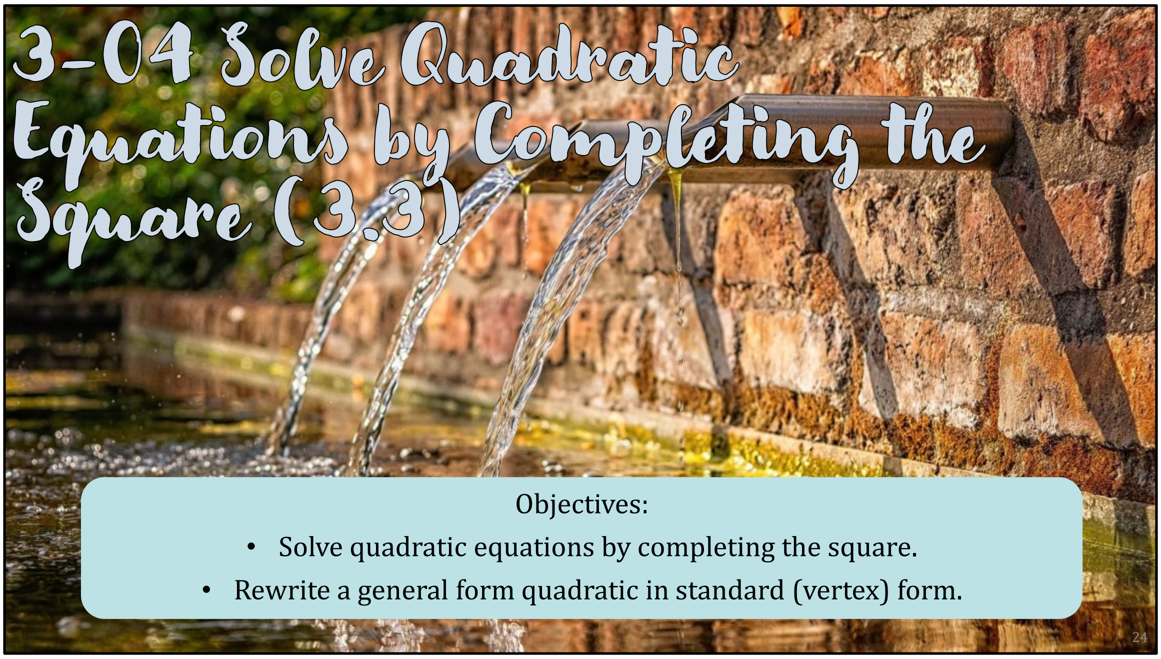
Where *x* is the number of times the price is dropped.

$$R = (3 - 0.5x)(20 + 5x) = 60 + 5x - 2.5x^2 = -2.5x^2 + 5x + 60$$

Maximum at vertex

$$x = -\frac{b}{2a}$$
$$x = -\frac{5}{2(-2.5)} = 1$$
$$R = -2.5(1)^2 + 5(1) + 60 = 62.50$$

They should lower the price one time to \$2.50 per pound to get a revenue of \$62.50


3-04 Solve Quadratic Equations by Completing the Square (3.3)
Objectives:
• Solve quadratic equations by completing the square.
• Rewrite a general form quadratic in standard (vertex) form.
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3-04 Solve Quadratic Equations by Completing the Square (3.3)

- The Perfect Square
- $(x+3)^2$

- $(x+k)^2 = x^2 + 2kx + k^2$
- $= ax^2 + bx + c$
- In a perfect square,

$$c = \left(\frac{b}{2}\right)^2$$

$=(x + 3)(x + 3)$
$= x^2 + 2(3x) + 3^2$
$= x^2 + 6x + 9$

3-04 Solve Quadratic Equations by Completing the Square (3.3)

- Complete the square and then factor.
    - $x^2 + 8x$

Add $\left(\frac{8}{2}\right)^2 = 4^2 = 16$ to get a perfect square or $(x + 4)^2$

3-04 Solve Quadratic Equations by Completing the Square (3.3)

Solve by Completing the Square

1. Rewrite the quadratic so *x* terms on one side and constant on other.
2. If the leading coefficient is not 1, divide everything by it.
3. Complete the square: add $\left(\frac{b}{2}\right)^2$ to both sides.
4. Rewrite the left hand side as a square (factor)
5. Square root both sides
6. Solve

- $x^2 + 6x = 16$

It already is

$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 16 + \left(\frac{6}{2}\right)^2$

$$x^2 + 6x + 32 = 16 + 9$$
$$(x + 3)^2 = 25$$
$$x + 3 = \pm 5$$
$$x = -3 \pm 5$$
$$x = 2, -8$$

3-04 Solve Quadratic Equations by Completing the Square (3.3)

- Solve $x^2 - 18x + 5 = 0$

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$$x^2 - 18x + 5 = 0$$
$$x^2 - 18x = -5$$
$$x^2 - 18x + 9^2 = -5 + 9^2$$
$$(x - 9)^2 = 76$$
$$x - 9 = \pm\sqrt{76}$$
$$x = 9 \pm 2\sqrt{19}$$

3-04 Solve Quadratic Equations by Completing the Square (3.3)

- Solve $2x^2 - 11x + 12 = 0$

$$2x^2 - 11x = -12$$

$$x^2 - \frac{11}{2}x = -6$$

$$x^2 - \frac{11}{2}x + \left(-\frac{\frac{11}{2}}{2}\right)^2 = -6 + \left(-\frac{\frac{11}{2}}{2}\right)^2 \rightarrow \left(-\frac{\frac{11}{2}}{2}\right)^2 = \left(-\frac{11}{4}\right)^2 = \frac{121}{16}$$

$$\left(x - \frac{11}{4}\right)^2 = \frac{25}{16}$$

$$x - \frac{11}{4} = \pm\frac{5}{4}$$

$$x = \frac{11}{4} \pm \frac{5}{4} = \frac{11 \pm 5}{4} = 4, \frac{3}{2}$$

3-04 Solve Quadratic Equations by Completing the Square (3.3)

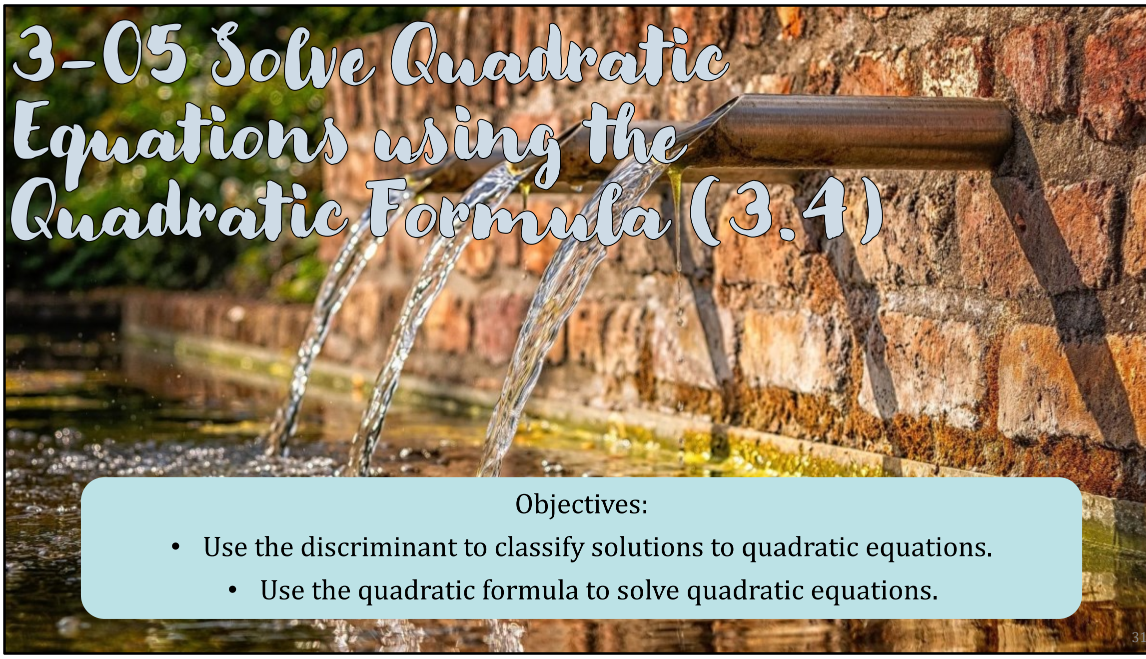
- **Writing quadratic functions in Standard Form**
- $y = a(x - h)^2 + k$
  - (h, k) is the vertex

1. Start with general form
2. Group the terms with the x
3. Factor out any number in front of the x^2
4. Add $\left(\frac{b}{2}\right)^2$ to both sides (inside the group on the right)
5. Rewrite as a perfect square
6. Subtract to get the y by itself

- $y = 2x^2 + 12x + 16$

2. $y = (2x^2 + 12x) + 16$
3. $y = 2(x^2 + 6x) + 16$
4. $y + 2(9) = 2(x^2 + 6x + 9) + 16$
5. $y + 18 = 2(x + 3)^2 + 16$
6. $y = 2(x + 3)^2 - 2$

Vertex is at (−3, −2)
−2 is the minimum for this function
Find the max or min by completing the square to find the vertex



3-05 Solve Quadratic Equations using the Quadratic Formula (3.4)

Objectives:

- Use the discriminant to classify solutions to quadratic equations.
- Use the quadratic formula to solve quadratic equations.

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3-05 Solve Quadratic Equations using the Quadratic Formula (3.4)

- Work with a Partner:
  Solve $ax^2 + bx + c = 0$
- $ax^2 + bx = -c$
- $x^2 + \frac{b}{a}x = -\frac{c}{a}$
- $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$
- $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$
- $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$
- $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$
- $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$
- $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$ax^2 + bx + c = 0$

$x^2 + \left(\frac{b}{a}\right)x + \left(\frac{c}{a}\right) = 0$ Divide by a to get a = 1

$x^2 + \left(\frac{b}{a}\right)x = -\left(\frac{c}{a}\right)$ Add –(c/a) to get x's by self

$x^2 + \left(\frac{b}{a}\right)x + \left(\frac{b}{2a}\right)^2 = -\left(\frac{c}{a}\right) + \left(\frac{b}{2a}\right)^2$ Add the square of half of middle to get perfect square

$\left(x + \left(\frac{b}{2a}\right)\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$ Factor left

$x + \left(\frac{b}{2a}\right) = \pm\sqrt{\frac{-4ac+b^2}{4a^2}}$ Square root

$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2-4ac}{4a^2}}$ Subtract

$x = \frac{\left(-b\pm\sqrt{b^2-4ac}\right)}{2a}$ Simplify

3-05 Solve Quadratic Equations using the Quadratic Formula (3.4)

- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- This is called the **quadratic formula** and always works for quadratic equations.
- The part under the square root, **discriminant**, tells you what kind of solutions you are going to have.
  - $b^2 - 4ac > 0$ → two distinct real solutions
  - $b^2 - 4ac = 0$ → exactly one real solution (a double solution)
  - $b^2 - 4ac < 0$ → two distinct imaginary solutions

3-05 Solve Quadratic Equations using the Quadratic Formula (3.4)

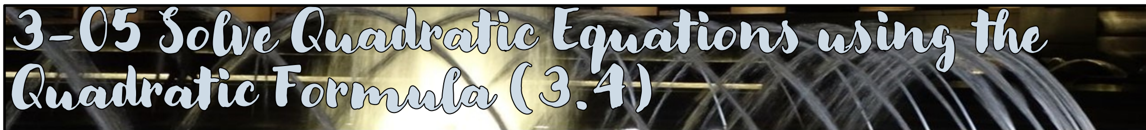
- What types of solutions to $5x^2 + 3x - 4 = 0$?
- Solve $5x^2 + 3x = 4$

$b^2 - 4ac = 3^2 - 4(5)(-4) = 9 + 80 = 89$ → two distinct real roots

Put in standard form → $5x^2 + 3x - 4 = 0$

Quadratic formula → $x = \frac{\left(-3 \pm \sqrt{32 - 4(5)(-4)}\right)}{2(5)}$

Simplify → $\frac{-3 \pm \sqrt{89}}{10}$



3-05 Solve Quadratic Equations using the Quadratic Formula (3.4)

- Solve $4x^2 - 6x + 3 = 0$

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$$4x^2 - 6x + 3 = 0$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{6 \pm \sqrt{36 - 48}}{8}$$

$$x = \frac{6 \pm \sqrt{-12}}{8}$$

$$x = \frac{6 \pm 2\sqrt{3}i}{8}$$

$$x = \frac{3 \pm \sqrt{3}i}{4}\ (reduce\ top\ and\ bottom\ by\ 2)$$

3-05 Solve Quadratic Equations using the Quadratic Formula (3.4)

- Find a possible pair of integer values for a and c so that the equation $ax^2 - 12x + c = 0$ has the given number and type of solution(s). Then write the equation.
- a. one real solution
- b. two imaginary solutions

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$$ax^2 - 12x + c = 0$$

Discriminant with 1 real solution

$$b^2 - 4ac = 0$$
$$(-12)^2 - 4ac = 0$$
$$144 = 4ac$$
$$36 = ac$$

Sample Answer: $a = 9, c = 4 \rightarrow 9x^2 - 12x + 4 = 0$

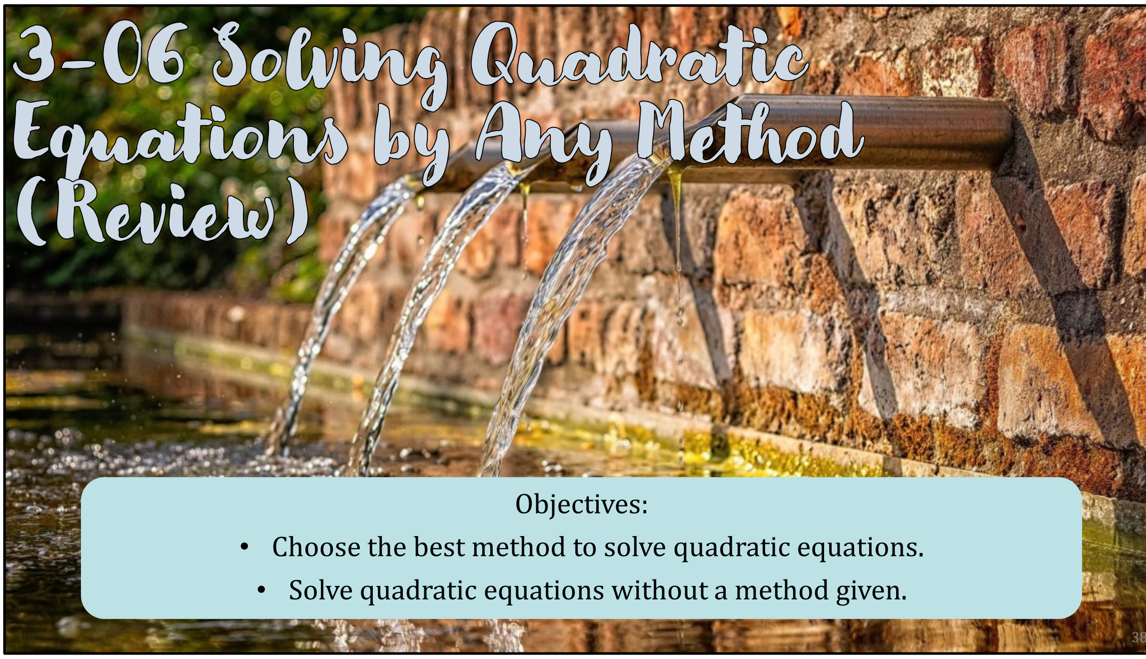
Discriminant with two imaginary solutions

$$b^2 - 4ac < 0$$
$$(-12)^2 - 4ac < 0$$
$$144 < 4ac$$
$$36 < ac$$

Sample Answer: $a = 10, c = 5 \rightarrow 10x^2 - 12x + 5 = 0$

3-05 Solve Quadratic Equations using the Quadratic Formula (3.4)

- **Real life problems**
- The height of an object that is hit or thrown up or down can be modeled by
- $h(t) = -16t^2 + v_0 t + s_0$
- where v_0 is the initial velocity (up +, down −), and s_0 is the initial height


3-06 Solving Quadratic Equations by Any Method (Review)
Objectives:
• Choose the best method to solve quadratic equations.
• Solve quadratic equations without a method given.
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3-06 Solving Quadratic Equations by Any Method (Review)

- **Choose the Best Method to Solve a Quadratic Equation**

To most efficiently solve a quadratic equation,

1. If x appears only once and it is squared—either x^2 or $(x - k)^2$— solve by taking square roots.
2. If both x^2 and x appear, make the equation equal to zero and…
    a. Try solving by factoring.
    b. If it cannot be factored quickly, solve by completing the square or the quadratic formula.
    c. Graphing is usually only as a last resort for complicated problems.

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3-06 Solving Quadratic Equations by Any Method (Review)

- Solve
- $x^2 + 6x + 5 = 0$
- $3x^2 - 12 = 5x$

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$$x^2 + 6x + 5 = 0$$

Factor

$$(x + 5)(x + 1) = 0$$
$$x + 5 = 0 \text{ or } x + 1 = 0$$
$$x = -5 \text{ or } -1$$

$$3x^2 - 12 = 5x$$
$$3x^2 - 5x - 12 = 0$$

Factor

$$(3x + 4)(x - 3) = 0$$
$$3x + 4 = 0 \text{ or } x - 3 = 0$$
$$x = -\frac{4}{3} \text{ or } 3$$

3-06 Solving Quadratic Equations by Any Method (Review)

- $4x^2 = 375 - x^2$
- $x^2 + 5x - 7 = 0$

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$$4x^2 = 375 - x^2$$
$$5x^2 = 375$$

Square roots

$$x^2 = 75$$
$$x = \pm 5\sqrt{3}$$

$$x^2 + 5x - 7 = 0$$

Try factoring (doesn't work)
Use quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-7)}}{2(1)}$$
$$x = \frac{-5 \pm \sqrt{53}}{2}$$

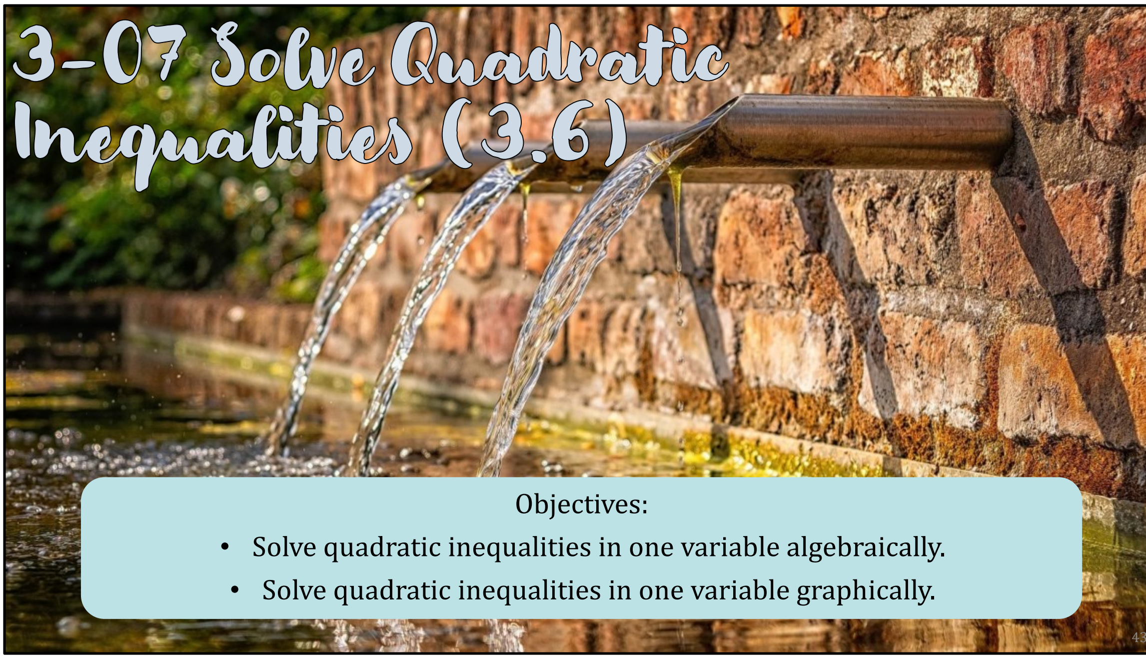
3-06 Solving Quadratic Equations by Any Method (Review)

- $3x^2 = 54x$

$$3x^2 = 54x$$

Factor

$$3x^2 - 54x = 0$$
$$3x(x - 18) = 0$$
$$x = 0 \text{ or } x - 18 = 0$$
$$x = 0 \text{ or } 18$$


3-07 Solve Quadratic Inequalities (3.6)
Objectives:
• Solve quadratic inequalities in one variable algebraically.
• Solve quadratic inequalities in one variable graphically.
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3-07 Solve Quadratic Inequalities (3.6)

- **Solve inequalities in one variable.**

1. Make = 0
2. Factor or use the quadratic formula to find the zeros
3. Graph the zeros on a number line (notice it cuts the line into three parts)
4. Pick a number in each of the three parts as test points
5. Test the points in the original inequality to see true or false
6. Write inequalities for the regions that were true

- $p^2 - 4p \le 5$

1. $p^2 - 4p - 5 \le 0$
2. $(p - 5)(p + 1) \le 0$

Zeros are 5, −1

4. test points = −2, 0, 6

$-2 \rightarrow 4 - 4(-2) \le 5 \rightarrow 12 \le 5$ false

$0 \rightarrow 0 - 4(0) \le 5 \rightarrow 0 \le 5$ true

$6 \rightarrow 36 - 4(6) \le 5 \rightarrow 12 \le 5$ false

6. middle region true so $-1 \le p \le 5$

3-07 Solve Quadratic Inequalities (3.6)

- Solve $x^2 - 4x > 45$

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$$x^2 - 4x > 45$$
$$x^2 - 4x - 45 > 0$$

Factor

$$(x + 5)(x - 9) > 0$$

Zeros

$$x + 5 = 0 \text{ or } x - 9 = 0$$
$$x = -5 \text{ or } 9$$

Graph and pick test points (−6, 0, 10)

Test points

−6: $(-6)^2 - 4(-6) > 45 \rightarrow 60 > 45$ true

0: $(0)^2 - 4(0) > 45 \rightarrow 0 > 45$ false

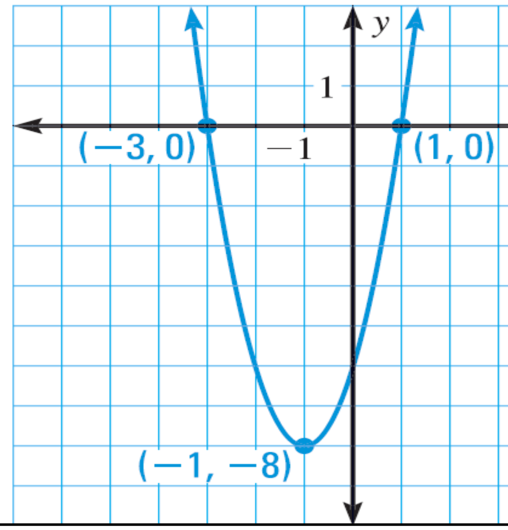
10: $(10)^2 - 4(10) > 45 \rightarrow 60 > 45$ true

Solution

$$x < -5 \text{ or } x > 9$$

3-07 Solve Quadratic Inequalities (3.6)

- Or you could also solve the quadratic inequality in one variable by graphing the quadratic
  1. Make the inequality = 0
  2. Plot points on a number line
  3. Quick graph
     a. When the graph is below the x-axis; ≤ 0
     b. When the graph is above the x-axis; ≥ 0

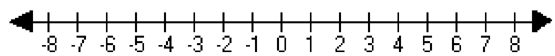


$2x^2 + 4x - 6 \leq 0$ when $-3 \leq x \leq 1$

$2x^2 + 4x - 6 \geq 0$ when $x \leq -3$ or $x \geq 1$

3-07 Solve Quadratic Inequalities (3.6)

- Solve using a graph.
  $$x^2 + x - 20 > 0$$



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$$x^2 + x - 20 > 0$$

Factor

$$(x+5)(x-4) > 0$$

Zeros

$$x + 5 = 0 \text{ or } x - 4 = 0$$
$$x = -5 \text{ or } x = 4$$

Plot the zeros

Sketch quick graph of $x^2 + x - 20$

Opens up

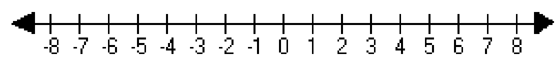
Because > 0, choose the *x*-values where the graph is above the *x*-axis.

$$x < -5 \text{ or } x > 4$$

3-07 Solve Quadratic Inequalities (3.6)

- Solve using a graph.

$$-2x^2 - 9x - 4 \geq 0$$



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$$-2x^2 - 9x - 4 \geq 0$$

Factor

$$(2x + 1)(-x - 4) \geq 0$$

Zeros

$$2x + 1 = 0 \text{ or } -x - 4 = 0$$

$$x = -\frac{1}{2} \text{ or } x = -4$$

Plot the zeros

Sketch quick graph of $-2x^2 - 9x - 4$

Opens down

Because ≥ 0, choose the *x*-values where the graph is above the *x*-axis.

$$-4 \leq x \leq -\frac{1}{2}$$